

# GENERALIZATION OF VELOCITY AND ACCELERATION TENSOR/VECTOR IN PARABOLOIDAL COORDINATES BASED ON RIEMANNIAN GEOMETRY AND GREAT METRIC TENSOR

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## ABSTRACT:

It is well known that Euclidean metric tensor is the foundation on which theoretical physics and mathematics are built upon even before now. But with the discovery of a great metric tensor in Spherical polar coordinates in all gravitational fields in nature has made Riemannian Geometry to be unlocked for exploration and exploitation in theoretical physics and mathematics. In this paper, we are out to obtain the generalized velocity and acceleration tensor/vector in Paraboloidal Coordinate based upon the great metric tensor.

Keywords: Riemannian geometry, Great metric tensor, velocity, acceleration and Paraboloidal Coordinates.

## 1:0 INTRODUCTION

Euclidean geometry has been the basic foundation on which all geometrical quantities in all orthogonal curvilinear coordinates in theoretical physics and mathematics are built upon. This geometry has a wide application in physics and mathematics because it has a well-developed metric tensor called the Euclidean metric tensor. In the year 1854, Georg Friedrich Bernhard Riemann (1826-1866) published his theory of Geometry and corresponding Tensorial Theory of Classical Mechanics in the Gravitational Field [1]. But those theories have never been completely understood because their quantities and equations are formulated in terms of three metric tensors for all gravitational fields in nature. And all the metric tensors for all gravitational fields in nature have hitherto not been fully exposed which make the Riemannian Geometry not to be opened up for exploration and exploitation in theoretical physics and mathematics.

It is most interesting and instructive note that the one and only one mathematically most simple and physically most natural and satisfactory metric tensor called the Great Metric Tensor for all gravitational fields in nature was discovered by Prof. S.X.K Howusu, in the year 2009 in his book entitled: Riemannian Revolution in Mathematics and Physics [2]. This metric

tensor has made Riemannian Geometry to be opened up for application in theoretical physics and mathematics. In view of this metric tensor called the Great Metric Tensor, we had been able to formulate some geometrical quantities based upon the Riemannian geometry in our previous publications [3,4,5]. Therefore in this paper, we carry out natural extensions or generalizations of the velocity and acceleration tensor/vector in Paraboloidal Coordinates for applications in theoretical physics and other related fields.

## 2:0 THEORY

The Cartesian coordinates  $(x,y,z,x^0)$  are defined in terms of the Paraboloidal Coordinates  $(u,v,\phi,x^0)$  by [6,7]:

$$x = uv\cos\phi \tag{1}$$

$$y = uv\sin\phi \tag{2}$$

$$z = \frac{1}{2}(u^2 - v^2) \tag{3}$$

Here:

$$r = \left[ \frac{1}{2}u^2v^2 + \frac{1}{4}(u^4 + v^4) \right]^{\frac{1}{2}} \tag{4}$$

and

$$\theta = \cos^{-1} \left\{ \frac{\frac{1}{2}(u^2 - v^2)}{\left[ \frac{1}{2}u^2v^2 + \frac{1}{4}(u^4 + v^4) \right]^{\frac{1}{2}}} \right\} \tag{5}$$

And

$$\phi = \phi \tag{6}$$

This great metric tensor for all gravitational fields in nature in spherical polar coordinates  $(u,v,\phi,x^0)$  is given as [2];

$$g_{00} = - \left( 1 + \frac{2}{c^2}f \right) \tag{7}$$

$$g_{11} = \left( 1 + \frac{2}{c^2}f \right)^{-1} \tag{8}$$

$$g_{22} = r^2 \tag{9}$$

$$g_{33} = r^2 \sin^2\theta \tag{10}$$

$$g_{\mu\nu} = 0; \text{ otherwise} \tag{11}$$

From the well-known transformation equation given by the covariant tensor [8] and consequently, upon transformation by using (7) – (10), we obtained the Riemannian metric tensor for all gravitational fields in the Paraboloidal Coordinates explicitly as:

$$g_{\mu\nu} = h_{\mu\nu} + f_{\mu\nu} \tag{12}$$

Where

$$h_{11} = h_{22} = u^2 + v^2 \tag{13}$$

$$h_{33} = u^2 v^2 \tag{14}$$

$$f_{11} = u^2 \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \tag{15}$$

$$f_{12} = uv \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \tag{16}$$

$$f_{22} = v^2 \sum_{n=1}^{\infty} \binom{-1}{n} \left(\frac{2}{c^2}\right)^n f^n \tag{17}$$

$$h_{00} = -1 \tag{18}$$

$$f_{00} = -\frac{2}{c^2} f^n \tag{19}$$

And

$$h_{\mu\nu} = 0 = f_{\mu\nu}; \text{ otherwise} \tag{20}$$

And the determinant of this metric tensor, denoted as g is given by;

$$g = -1 \tag{21}$$

If  $g_{\mu\nu}$  is a covariant metric tensor, then according to tensor analysis the contravariant metric tensor for this Riemannian metric tensor denoted as  $g^{\mu\nu}$  is given as;

$$g^{00} = -\left(1 + \frac{2}{c^2} f\right)^{-1} \tag{22}$$

$$g^{11} = \frac{1}{(u^2 + v^2)^2} \left[ u^2 \left(1 + \frac{2}{c^2} f\right) + v^2 \right] \tag{23}$$

$$g^{12} = g^{21} = \frac{-2uv}{c^2(u^2 + v^2)^2} f \tag{24}$$

$$g^{22} = \frac{1}{(u^2 + v^2)^2} \left[ v^2 \left( 1 + \frac{2}{c^2} f \right) + u^2 \right] \quad (25)$$

$$g^{33} = \frac{1}{u^2 v^2} \quad (26)$$

And

$$g^{\mu\nu} = 0 ; \textit{otherwise} \quad (27)$$

These metric tensors define Riemannian volume element, Riemannian gradient, Riemannian line element, Riemannian curl, Riemannian divergence and Riemannian Laplacian in Paraboloidal Coordinates, according to the theory of tensor and vector analysis []. These quantities are necessary and sufficient for the derivation of the fields in all Paraboloidal distribution of charges, current and mass for the derivation of the equation of motion for test particle on all gravitational fields, we shall derive the expression for Riemannian velocity and acceleration in Parabolidal Coordinates.

## 2:1 THE GENERALIZED VELOCITY TENSOR/VECTOR IN PARABOLOIDAL COORDINATES

Based on the theory of tensor analysis, the linear velocity tensor in four dimensional space-time,  $u^\alpha$  is given in all gravitational fields in all orthogonal curvilinear coordinates  $x^u$  by [8]:

$$u^\alpha = \frac{d}{d\tau} x^\alpha = \dot{x}^\alpha \quad (28)$$

Where  $\tau$  proper time and a dot is denotes differentiation with respect to time.  $u^0, u^1, u^2$  and  $u^3$  are given as:

$$u^0 = c\dot{t} \quad (29)$$

$$u^1 = \dot{u} \quad (30)$$

$$u^2 = \dot{v} \quad (31)$$

$$u^3 = \dot{\phi} \quad (32)$$

The generalized velocity tensor according to the theory of tensor analysis, the coordinates  $(u, v, \phi)$  is given as:

$$\underline{u}_R = [u_u, u_v, u_\phi, u_{x^0}] \quad (33)$$

Where

$$u_{x^0} = -c \left( 1 + \frac{2}{c^2} \right)^{\frac{1}{2}} \dot{t} \quad (34)$$

$$u_u = \left[ v^2 + u^2 \left( 1 + \frac{2}{c^2} f \right)^{-1} \right]^{\frac{1}{2}} \dot{u} \quad (35)$$

$$u_v = \left[ u^2 + v^2 \left( 1 + \frac{2}{c^2} f \right)^{-1} \right]^{\frac{1}{2}} \dot{v} \quad (36)$$

And

$$u_\phi = uv\dot{\phi} \quad (37)$$

This is the generalized linear velocity vector.

## 2:2 THE GENERALIZED ACCELERATION TENSOR/VECTOR IN PARABOLOIDAL COORDINATES

The generalized linear acceleration tensor in 4-dimensional space,  $a_R^\alpha$  in gravitational fields in nature and all orthogonal curvilinear coordinates  $x^\alpha$  is obtained by theory of tensor analysis as [8]

$$a_R^\alpha = \ddot{x}^\alpha + \Gamma_{\mu\nu}^\alpha \dot{x}^\mu \dot{x}^\nu \quad (38)$$

Where  $\Gamma_{\mu\nu}^\alpha$  is the Christoffel symbol of the second kind (or coefficient of affine connection) pseudo tensor and a dot denotes one differentiation with respect to proper time,  $\tau$ . The non-zero results of  $\Gamma_{\mu\nu}^\alpha$  based upon the great metric tensor in paraboloidal coordinates are given as;

$$\Gamma_{00}^0 = \frac{1}{c^2} \left( 1 + \frac{2}{c^2} f \right)^{-1} f_{,0} \quad (39)$$

$$\Gamma_{01}^0 = \Gamma_{10}^0 = \frac{1}{c^2} \left( 1 + \frac{2}{c^2} f \right)^{-1} f_{,1} \quad (40)$$

$$\Gamma_{02}^0 = \Gamma_{20}^0 = \frac{1}{c^2} \left( 1 + \frac{2}{c^2} f \right)^{-1} f_{,2} \quad (41)$$

$$\Gamma_{03}^0 = \Gamma_{30}^0 = \frac{1}{c^2} \left( 1 + \frac{2}{c^2} f \right)^{-1} f_{,3} \quad (42)$$

$$\Gamma_{11}^0 = \frac{-u^2}{c^2} \left( 1 + \frac{2}{c^2} f \right)^{-3} f_{,0} \quad (43)$$

$$\Gamma_{12}^0 = \Gamma_{21}^0 = \frac{-uv}{c^2} \left( 1 + \frac{2}{c^2} f \right)^{-3} f_{,0} \quad (44)$$

$$\Gamma_{22}^0 = \frac{-v^2}{c^2} \left( 1 + \frac{2}{c^2} f \right)^{-3} f_{,0} \quad (45)$$

$$\Gamma_{00}^1 = \frac{1}{c^2(u^2 + v^2)^2} \left[ \left( u^2 \left( 1 + \frac{2}{c^2} f \right) + v^2 \right) f_{,1} + \frac{2uv}{c^2} f f_{,2} \right] \quad (46)$$

$$\Gamma_{01}^1 = \Gamma_{10}^1 = \frac{u^2}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (v^2 - u^2) - (v^2 + u^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,0} \quad (47)$$

$$\Gamma_{02}^1 = \Gamma_{20}^1 = \frac{uv}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (v^2 - u^2) - (v^2 + u^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,0} \quad (48)$$

$$\Gamma_{11}^1 = \frac{u^2}{c^2(u^2 + v^2)^2} \left[ \left[ v^2 f - u^2 \left( 1 + \frac{2}{c^2} f \right) - v^2 \right] f_{,1} + 2uvf f_{,2} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} \quad (49)$$

$$\Gamma_{12}^1 = \Gamma_{21}^1 = \frac{u}{c^2(u^2 + v^2)^2} \left[ \frac{2v^3}{c^2} f f_{,1} - u \left[ u^2 \left( 1 + \frac{2}{c^2} \right) + v^2 \right] f_{,2} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} \quad (50)$$

$$\Gamma_{13}^1 = \Gamma_{31}^1 = \frac{u}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (v^2 - u^2) - (v^2 + u^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \quad (51)$$

$$\Gamma_{22}^1 = \frac{v}{c^2(u^2 + v^2)^2} \left[ \left[ u^2 v \left( 1 + \frac{2}{c^2} f \right) + v^3 \right] f_{,1} + \left[ \frac{2uv^2}{c^2} f - 2u^3 \left( 1 + \frac{2}{c^2} f \right) - 2v^2 u \right] f_{,2} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} \quad (52)$$

$$\Gamma_{23}^1 = \Gamma_{32}^1 = \frac{uv}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (v^2 - u^2) - (v^2 + u^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \quad (53)$$

$$\Gamma_{33}^1 = \frac{-uv^2}{(u^2 + v^2)^2} \left[ (u^2 + v^2) + \frac{4u^2}{c^2} f \right] \quad (54)$$

$$\Gamma_{00}^2 = \frac{-1}{c^2(u^2 + v^2)^2} \left[ 2uvf f_{,1} - \left[ v^2 \left( 1 + \frac{2}{c^2} f \right) + u^2 \right] f_{,2} \right] \quad (55)$$

$$\Gamma_{01}^2 = \Gamma_{10}^2 = \frac{-uv}{c^2(u^2 + v^2)^2} (v^2 + u^2) \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,0} \quad (56)$$

$$\Gamma_{02}^2 = \Gamma_{20}^2 = \frac{v^2}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (u^2 - v^2) - (u^2 + v^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,0} \quad (57)$$

$$\Gamma_{11}^2 = \frac{u}{c^2(u^2 + v^2)^2} \left[ \left[ \frac{2vu^2}{c^2} f - 2v^3 \left( 1 + \frac{2}{c^2} f \right) - 2vu^2 \right] f_{,1} + \left[ u^3 + uv^2 \left( 1 + \frac{2}{c^2} f \right) \right] f_{,2} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} \quad (58)$$

$$\Gamma_{12}^2 = \Gamma_{21}^2 = \frac{2v}{c^2(u^2 + v^2)^2} \left[ \frac{u^3}{c^2} f_{,2} - v \left[ v^2 \left( 1 + \frac{2}{c^2} f \right) + u^2 \right] f_{,1} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} \quad (59)$$

$$\Gamma_{13}^2 = \Gamma_{31}^2 = \frac{uv}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (u^2 - v^2) - (u^2 + v^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \quad (60)$$

$$\Gamma_{22}^2 = \frac{-v^2}{c^2(u^2 + v^2)^2} \left[ \frac{2uv}{c^2} f f_{,1} - \left[ \frac{2f}{c^2} (u^2 - v^2) - (u^2 + v^2) \right] f_{,2} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} \quad (61)$$

$$\Gamma_{23}^2 = \Gamma_{32}^2 = \frac{v^2}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (u^2 - v^2) - (u^2 + v^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \quad (62)$$

$$\Gamma_{33}^2 = \frac{-u^2 v}{(u^2 + v^2)^2} \quad (63)$$

$$\Gamma_{00}^3 = \frac{1}{c^2 u^2 v^2} f_{,3} \quad (64)$$

$$\Gamma_{11}^3 = \frac{1}{c^2 v^2} \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \quad (65)$$

$$\Gamma_{12}^3 = \Gamma_{21}^3 = \frac{1}{c^2 uv} \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \quad (66)$$

$$\Gamma_{13}^3 = \Gamma_{31}^3 = \frac{1}{u} \quad (67)$$

$$\Gamma_{22}^3 = \frac{1}{c^2 u^2 (u^2 + v^2)^2} f_{,3} \quad (68)$$

$$\Gamma_{23}^3 = \Gamma_{32}^3 = \frac{1}{v} \quad (69)$$

$$\Gamma_{\mu\nu}^\alpha = 0 ; \textit{otherwise} \quad (70)$$

It follows from (39) – (70) that;

$$a_R^0 = c\ddot{t} + \left( 1 + \frac{2}{c^2} f \right)^{-1} f_{,0}(t)^2 + \frac{2}{c} \left( 1 + \frac{2}{c^2} f \right)^{-1} f_{,1}t\dot{u} + \frac{2}{c} \left( 1 + \frac{2}{c^2} f \right)^{-1} f_{,2}t\dot{v} + \frac{2}{c} \left( 1 + \frac{2}{c^2} f \right)^{-1} f_{,3}t\dot{\phi} - \frac{u^2}{c^2} \left( 1 + \frac{2}{c^2} f \right)^{-3} f_{,0}(\dot{u})^2 - \frac{2uv}{c^2} \left( 1 + \frac{2}{c^2} f \right)^{-3} f_{,0}\dot{u}\dot{v} - \frac{v^2}{c^2} \left( 1 + \frac{2}{c^2} f \right)^{-3} f_{,0}(\dot{v})^2 \quad (71)$$

And

$$\begin{aligned}
 a_R^1 = \ddot{u} + \frac{1}{(u^2 + v^2)^2} & \left[ \left[ u^2 \left( 1 + \frac{2}{c^2} f \right) + v^2 \right] f_{,1} + \frac{2uv}{c^2} f f_{,2} \right] (\dot{t})^2 \\
 & + \frac{2u^2}{c(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (v^2 - u^2) - (v^2 + u^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,0} \dot{t} \dot{u} \\
 & + \frac{2uv}{c(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (v^2 - u^2) - (v^2 + u^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,0} \dot{t} \dot{v} \\
 & + \frac{u^2}{c^2(u^2 + v^2)^2} \left[ \left[ v^2 f - u^2 \left( 1 + \frac{2}{c^2} f \right) - v^2 \right] f_{,1} \right. \\
 & \left. + 2uv f f_{,2} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} (\dot{u})^2 \\
 & + \frac{2u}{c^2(u^2 + v^2)^2} \left[ \frac{2v^3}{c^2} f f_{,1} - u \left[ u^2 \left( 1 + \frac{2}{c^2} f \right) + v^2 \right] f_{,2} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} \dot{u} \dot{v} \\
 & + \frac{2u^2}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (v^2 - u^2) - (v^2 + u^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \dot{u} \dot{\phi} \\
 & + \frac{v}{c^2(u^2 + v^2)^2} \left[ \left[ u^2 v \left( 1 + \frac{2}{c^2} f \right) + v^3 \right] f_{,1} \right. \\
 & \left. + \left[ \frac{2uv^3}{c^2} f - 2u^3 \left( 1 + \frac{2}{c^2} f \right) - 2uv^2 \right] f_{,2} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} (\dot{v})^2 \\
 & + \frac{2uv}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (v^2 - u^2) - (v^2 + u^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \dot{v} \dot{\phi} \\
 & + \frac{(-1)uv^2}{(u^2 + v^2)^2} \left[ (u^2 + v^2) + \frac{4u^2}{c^2} f \right] (\dot{\phi})^2
 \end{aligned} \tag{72}$$

And



$$\begin{aligned}
 & a_R^2 \\
 &= \ddot{v} + \frac{-1}{(u^2 + v^2)^2} \left[ 2uvf f_{,1} - \left[ v^2 \left( 1 + \frac{2}{c^2} f \right) + u^2 \right] f_{,2} \right] (\dot{t})^2 \\
 &+ \frac{2(-1)uv}{c(u^2 + v^2)^2} (v^2 + u^2) \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,0} \dot{t} \dot{u} \\
 &+ \frac{2v^2}{c(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (u^2 - v^2) - (u^2 + v^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,0} \dot{t} \dot{v} \\
 &+ \frac{u}{c^2(u^2 + v^2)^2} \left[ \left[ \frac{2u^2v}{c^2} f - 2v^3 \left( 1 + \frac{2}{c^2} f \right) - 2u^2v \right] f_{,1} \right. \\
 &+ \left. \left[ u^3 + uv^2 \left( 1 + \frac{2}{c^2} f \right) \right] f_{,2} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} (\dot{u})^2 \\
 &+ \frac{4v}{c^2(u^2 + v^2)^2} \left[ \frac{u^3}{c^2} f_{,2} - v \left[ v^2 \left( 1 + \frac{2}{c^2} f \right) + u^2 \right] f_{,1} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} \dot{u} \dot{v} \\
 &+ \frac{2uv}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (u^2 - v^2) - (u^2 + v^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \dot{u} \dot{\phi} \\
 &+ \frac{(-1)v^2}{c^2(u^2 + v^2)^2} \left[ \frac{2uv}{c^2} f_{,1} - \left[ \frac{2f}{c^2} (u^2 - v^2) - (u^2 + v^2) \right] f_{,2} \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} (\dot{v})^2 \\
 &+ \frac{2v^2}{c^2(u^2 + v^2)^2} \left[ \frac{2f}{c^2} (u^2 - v^2) - (u^2 + v^2) \right] \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \dot{v} \dot{\phi} \\
 &+ \frac{(-1)u^2v}{(u^2 + v^2)^2} (\dot{\phi})^2
 \end{aligned} \tag{73}$$

And

$$\begin{aligned}
 a_R^3 = \ddot{\phi} + \frac{1}{u^2 v^2} f_{,3} (\dot{t})^2 + \frac{1}{c^2 v^2} \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} (\dot{u})^2 + \frac{2}{c^2 uv} \left( 1 + \frac{2}{c^2} f \right)^{-2} f_{,3} \dot{u} \dot{v} + \frac{2}{u} \dot{u} \dot{\phi} \\
 + \frac{1}{c^2 u^2 (u^2 + v^2)^2} f_{,3} (\dot{v})^2 + \frac{2}{v} \dot{v} \dot{\phi}
 \end{aligned} \tag{74}$$

Equation (71) – (74) is called the Riemann Linear Acceleration Tensor.

Hence, the

Great Riemann Acceleration Vector,  $\underline{a}_R$  is given as;

$$\underline{a}_R = [(a_R)_u, (a_R)_v, (a_R)_\phi, (a_R)_{x^0}] \tag{75}$$

Where

$$(a_R)_{x^0} = (g_{00})^{\frac{1}{2}} [a_R^0] \tag{76}$$

$$(a_R)_u = (g_{11})^{\frac{1}{2}} [a_R^1] \tag{77}$$

$$(a_R)_v = (g_{22})^{\frac{1}{2}} [a_R^2] \tag{78}$$

$$(a_R)_\emptyset = (g_{33})^{\frac{1}{2}}[a_R^3] \quad (79)$$

Equation (76) – (79) is called the Great Riemannian Laplacian Acceleration Vector for all gravitational field in nature in Paraboloidal Coordinates.

### 3:0 RESULTS AND DISCUSSIONS

In this paper, we obtained component of the Great Riemannian Linear velocity tensor/vector and the Great Riemannian Linear Acceleration tensor/vector in Paraboloidal Coordinates as (34) – (37) and (76) – (79) respectively.

These results derived in this paper are sufficient and necessary for expressing all Riemannian mechanical quantities in all gravitational fields in nature (Riemannian Hamiltonian, Riemannian Lagrangian, Riemannian linear momentum and Riemannian kinetic energy) in terms of Paraboloidal Coordinates.

### 4:0 CONCLUSIONS

The Great Riemannian Linear Velocity vector and the Great Riemannian Linear Acceleration vector obtain in this paper in (34) – (37) and (76) – (79) respectively pave a way for expressing all Riemannian dynamic laws of motion (Newton's law, Hamilton's law, Lagrange's law, Einstein's special Relativistic law of motion and Schrodinger's law of quantum mechanics) in terms of Paraboloidal Coordinates.

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